

## CHAPTER 3

# HEAT TRANSFER

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**H**HEAT transfer is energy in transit because of a temperature difference. The thermal energy is transferred from one region to another by three modes of heat transfer: **conduction**, **radiation**, and **convection**. Heat transfer is among a group of energy transport phenomena that includes mass transfer (see Chapter 5), momentum transfer (see Chapter 2), and electrical conduction. Transport phenomena have similar rate equations, in which flux is proportional to a potential difference. In heat transfer by conduction and convection, the potential difference is the temperature difference. Heat, mass, and momentum transfer are often considered together because of their similarities and interrelationship in many common physical processes.

This chapter presents the elementary principles of single-phase heat transfer with emphasis on HVAC applications. Boiling and condensation are discussed in Chapter 4. More specific information on heat transfer to or from buildings or refrigerated spaces can be found in Chapters 25 through 32 of this volume and in Chapter 12 of the 2002 *ASHRAE Handbook—Refrigeration*. Physical properties of substances can be found in Chapters 18, 22, 24, and 39 of this volume and in Chapter 8 of the 2002 *ASHRAE Handbook—Refrigeration*. Heat transfer equipment, including evaporators, condensers, heating and cooling coils, furnaces, and radiators, is covered in the 2004 *ASHRAE Handbook—HVAC Systems and Equipment*. For further information on heat transfer, see the Bibliography.

### HEAT TRANSFER PROCESSES

In the applications considered here, the materials are assumed to behave as a **continuum**: that is, the smallest volume considered contains enough molecules so that thermodynamic properties (e.g., density) are valid. The smallest length dimension in most engineering applications is about 100  $\mu\text{m}$  ( $10^{-1}$  mm). At a standard pressure of 14.696 psi at 32°F, there are about  $3 \times 10^{10}$  molecules of air in a volume of  $10^{-3}$  mm<sup>3</sup>; even at a pressure of 0.0001 psi, there are  $3 \times 10^7$  molecules. With such a large number of molecules in a very small volume, the variation of the mass of gas in the volume resulting from variation in the number of molecules is extremely small, and the mass per unit volume at that location can be used as the density of the material at that point. Other thermodynamic properties (e.g., temperature) can also serve as point functions.

**Thermal Conduction.** This heat transfer mechanism transports energy between parts of a continuum by transfer of kinetic energy between particles or groups of particles at the atomic level. In gases, conduction is caused by elastic collision of molecules; in liquids and electrically nonconducting solids, it is believed to be caused by longitudinal oscillations of the lattice structure. Thermal conduction in metals occurs, like electrical conduction, through the motion of free electrons. Thermal energy transfer occurs in the direction of decreasing temperature. In opaque solid bodies, thermal conduction is the significant heat transfer mechanism because no net material flows in the process and radiation is not a factor.

Fourier's law for conduction is

$$q'' = -k \frac{\partial t}{\partial x} \quad (1a)$$

where

- $q''$  = heat flux (heat transfer rate per unit area  $A$ ), Btu/h·ft<sup>2</sup>
- $\partial t/\partial x$  = temperature gradient, °F/ft
- $k$  = thermal conductivity, Btu/h·ft<sup>2</sup>·°F

The magnitude of heat flux  $q''$  in the  $x$  direction is directly proportional to the temperature gradient  $\partial t/\partial x$ . The proportionality factor is thermal conductivity  $k$ . The minus sign indicates that heat flows in the direction of decreasing temperature. If temperature is steady, one-dimensional, and uniform over the surface, integrating Equation (1a) over area  $A$  yields

$$q = -kA \frac{\partial t}{\partial x} \quad (1b)$$

Equation (1b) applies where temperature is a function of  $x$  only. In the equation,  $q$  is the total heat transfer rate across the area of cross section  $A$  perpendicular to the  $x$  direction. Thermal conductivity values are sometimes given in other units, but consistent units must be used in Equation (1b).

If  $A$  (e.g., a slab wall) and  $k$  are constant, Equation (1b) can be integrated to yield

$$q = \frac{kA(t_1 - t_2)}{L} = \frac{t_1 - t_2}{L/(kA)} \quad (2)$$

where  $L$  is wall thickness,  $t_1$  is the temperature at  $x = 0$ , and  $t_2$  is the temperature at  $x = L$ .

**Thermal Radiation.** In conduction and convection, heat transfer takes place through matter. In thermal radiation, energy is emitted from a surface and transmitted as electromagnetic waves, and then absorbed by a receiving surface. Whereas conduction and convection heat transfer rates are driven primarily by temperature gradients and somewhat by temperature because of temperature-dependent properties, radiative heat transfer rates are driven by the fourth power of the absolute temperature and increase rapidly with temperatures. Unlike conduction and convection, no medium is required to transmit electromagnetic energy.

Every surface emits energy. The rate of emitted energy per unit area is the **emissive power** of the surface. At any given temperature, the emissive power depends on the surface characteristics. At a defined surface temperature, an ideal surface (perfect emitter) emits the highest amount of energy. Such a surface is also a perfect absorber (i.e., it absorbs all incident radiant energy) and is called a **blackbody**. The blackbody emissive power  $W_b$  is given by the Stefan-Boltzmann relation

$$W_b = \sigma T^4$$

The preparation of this chapter is assigned to TC 1.3, Heat Transfer and Fluid Flow.

where  $\sigma = 0.1712 \times 10^{-8}$  Btu/h·ft<sup>2</sup>·°R<sup>4</sup> is the Stefan-Boltzmann constant.  $W_b$  is in Btu/ft<sup>2</sup> and  $T$  is in °F.

The ratio of the emissive power of a nonblack surface to the blackbody emissive power at the same temperature is the surface **emissivity**  $\epsilon$ , which varies with wavelength for some surfaces (see discussion in the section on Actual Radiation). **Gray** surfaces are those for which radiative properties are wavelength-independent.

When a gray surface (area  $A_1$ , temperature  $T_1$ ) is completely enclosed by another gray surface (area  $A_2$ , temperature  $T_2$ ) and separated by a transparent gas, the net radiative heat transfer rate from surface 1 is

$$q_1 = \frac{A_1(W_{b1} - W_{b2})}{1/\epsilon_1 + A_1/A_2(1 - \epsilon_2)/\epsilon_2}$$

where  $W_b$  and  $\epsilon$  are the blackbody emissive power and emissivity of the surfaces. The **radiative heat transfer coefficient**  $h_r$  is defined as

$$h_r = \frac{q_1/A_1}{T_1 - T_2} = \frac{\sigma(T_1^2 + T_2^2)(T_1 + T_2)}{1/\epsilon_1 + A_1/A_2(1 - \epsilon_2)/\epsilon_2} \quad (3a)$$

For two common cases, Equation (3a) simplifies to

$$A_1 = A_2: \quad h_r = \frac{\sigma(T_1^2 + T_2^2)(T_1 + T_2)}{1/\epsilon_1 + 1/\epsilon_2 - 1} \quad (3b)$$

$$A_2 \gg A_1: \quad h_r = \sigma\epsilon_1(T_1^2 + T_2^2)(T_1 + T_2) \quad (3c)$$

Note that  $h_r$  is a function of the surface temperatures, one of which is often unknown.

**Thermal Convection.** When fluid flows are produced by external sources such as blowers and pumps, the solid-to-fluid heat transfer is called **forced convection**. If fluid flow is generated by density differences caused solely by temperature variation, the heat transfer is called **natural convection**. **Free convection** is sometimes used to denote natural convection in a semi-infinite fluid.

For convective heat transfer from a solid surface to an adjacent fluid, the **convective heat transfer coefficient**  $h$  is defined by

$$q'' = h(t_s - t_{ref})$$

where

- $q''$  = heat flux from solid surface to fluid
- $t_s$  = solid surface temperature, °F
- $t_{ref}$  = fluid reference temperature for defining convective heat transfer coefficient

If the convective heat transfer coefficient, surface temperature, and fluid temperature are uniform, integrating this equation over surface area  $A_s$  gives the total convective heat transfer rate from the surface:

$$q_c = hA_s(t_s - t_{ref}) = \frac{t_s - t_{ref}}{1/hA_s} \quad (4)$$

**Combined Heat Transfer Coefficient.** For natural convection from a surface to a surrounding gas, radiant and convective heat transfer rates are usually of comparable magnitudes. In such a case, the combined heat transfer coefficient is the sum of the two heat transfer coefficients. Thus,

$$h = h_c + h_r$$

where  $h$ ,  $h_c$ , and  $h_r$  are the combined, natural convection, and radiation heat transfer coefficients, respectively.

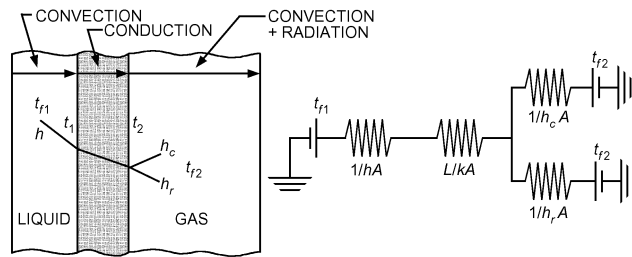


Fig. 1 Thermal Circuit

**Thermal Resistance R.** In Equation (2) for conduction in a slab, Equation (3a) for radiative heat transfer rate between two surfaces, and Equation (4) for convective heat transfer rate from a surface, the heat transfer rate can be expressed as a temperature difference divided by a thermal resistance  $R$ . Thermal resistance is analogous to electrical resistance, with temperature difference and heat transfer rate instead of potential difference and current, respectively. All the tools available for solving series electrical resistance circuits can also be applied to series heat transfer circuits. For example, consider the heat transfer rate from a liquid to the surrounding gas separated by a constant cross-sectional area solid, as shown in Figure 1. The heat transfer rate from the fluid to the adjacent surface is by convection, then across the solid body by conduction, and finally from the solid surface to the surroundings by convection and radiation, as shown in the figure. A series circuit using the equations for the heat transfer rates for each mode is also shown.

From the circuit, the heat transfer rate is computed as

$$q = \frac{t_{f1} - t_{f2}}{\frac{1}{hA} + \frac{L}{kA} + \frac{(1/h_c A)(1/h_r A)}{1/h_c A + 1/h_r A}}$$

For steady-state problems, thermal resistance can be used

- With several layers of materials having different thermal conductivities, if temperature distribution is one-dimensional
- With complex shapes for which exact analytical solutions are not available, if conduction shape factors are available
- In many problems involving combined conduction, convection, and radiation

Although the solutions are exact in series circuits, the solutions with parallel circuits are approximate because a one-dimensional temperature distribution is assumed.

Further use of the resistance concept is discussed in the sections on Thermal Conduction and Overall Heat Transfer Coefficient.

## THERMAL CONDUCTION

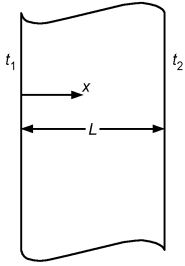
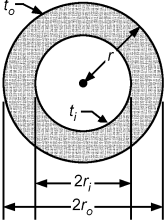
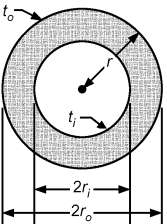
### Steady-State Conduction

**One-Dimensional Conduction.** Solutions to steady-state heat transfer rates in (1) a slab of constant cross-sectional area with parallel surfaces maintained at uniform but different temperatures, (2) a hollow cylinder with heat transfer across cylindrical surfaces only, and (3) a hollow sphere are given in Table 1.

Mathematical solutions to a number of more complex heat conduction problems are available in Carslaw and Jaeger (1959). Complex problems can also often be solved by graphical or numerical methods such as described by Adams and Rogers (1973), Croft and Lilley (1977), and Patankar (1980).

**Two- and Three-Dimensional Conduction: Shape Factors.** There are many steady cases with two- and three-dimensional temperature distribution where a quick estimate of the heat transfer rate

**Table 1 Heat Transfer Rate and Thermal Resistance for Sample Configurations**

Configuration	Heat Transfer Rate	Thermal Resistance
Constant cross-sectional area slab 	$q_x = kA_x \frac{t_1 - t_2}{L}$	$\frac{L}{kA_x}$
Hollow cylinder with negligible heat transfer from end surfaces 	$q_r = \frac{2\pi kL(t_i - t_o)}{\ln\left(\frac{r_o}{r_i}\right)}$	$R = \frac{\ln(r_o/r_i)}{2\pi kL}$
Hollow sphere 	$q_r = \frac{4\pi k(t_i - t_o)}{\frac{1}{r_i} + \frac{1}{r_o}}$	$R = \frac{1/r_i - 1/r_o}{4\pi k}$

is desired. **Conduction shape factors** provide a method for getting such estimates. Heat transfer rates obtained by using conduction shape factors are approximate because one-dimensional temperature distribution cannot be assumed in those cases. Using the conduction shape factor  $S$ , the heat transfer rate is expressed as

$$q = Sk(t_1 - t_2) \quad (5)$$

where  $k$  is the material's thermal conductivity, and  $t_1$  and  $t_2$  are the temperatures of two surfaces. Conduction shape factors for some common configurations are given in Table 2. When using a conduction shape factor, the thermal resistance is

$$R = 1/Sk \quad (6)$$

### Transient Conduction

Often, heat transfer and temperature distribution under transient (varying with time) conditions must be known. Examples are (1) cold-storage temperature variations on starting or stopping a refrigeration unit, (2) variation of external air temperature and solar irradiation affecting the heat load of a cold-storage room or wall temperatures, (3) the time required to freeze a given material under certain conditions in a storage room, (4) quick-freezing objects by direct immersion in brines, and (5) sudden heating or cooling of fluids and solids from one temperature to another.

For slabs of constant cross-sectional areas, cylinders, and spheres, analytical solutions in the form of infinite series are available. For solids with irregular boundaries, use numerical methods.

**Lumped Mass Analysis.** One elementary transient heat transfer model predicts the rate of temperature change of a body or material with uniform temperature, such as a well-stirred reservoir of fluid whose temperature is a function of time only and spatially uniform at all instants. Such an approximation is valid if

$$Bi = \frac{h(V/A_s)}{k} \leq 0.1$$

where

$Bi$  = Biot number [ratio of (1) internal temperature difference required to move energy within the solid of liquid to (2) temperature difference required to add or remove the same energy at the surface]

$h$  = surface heat transfer coefficient

$V$  = material's volume

$A_s$  = surface area exposed to convective heat transfer

$k$  = material's thermal conductivity

The temperature is given by

$$Mc_p \frac{dt}{d\tau} = q_{net} + q_{gen} \quad (7)$$

where

$M$  = body mass

$c_p$  = specific heat at constant pressure

$q_{gen}$  = internal heat generation

$q_{net}$  = net heat transfer rate to substance (into substance is positive, and out of substance is negative)

Equation (7) is applicable when pressure around the substance is constant; if the volume is constant, replace  $c_p$  with the constant-volume specific heat  $c_v$ . Note that with the density of solids and liquids being substantially constant, the two specific heats are almost equal. The term  $q_{net}$  may include heat transfer by conduction, convection, or radiation and is the difference between the heat transfer rates into and out of the body. The term  $q_{gen}$  may include a chemical reaction (e.g., curing concrete) or heat generation from a current passing through a metal.

A common case for lumped analysis is a solid body exposed to a fluid at a different temperature. The time taken for the solid temperature to change to  $t_f$  is given by

$$\ln \frac{t_f - t_\infty}{t_o - t_\infty} = -\frac{hA\tau}{Mc_p} \quad (8)$$

where

$M$  = mass of solid

$c_p$  = specific heat of solid

$A$  = surface area of solid

$h$  = surface heat transfer coefficient

$\tau$  = time required for temperature change

$t_f$  = final solid temperature

$t_o$  = initial uniform solid temperature

$t_\infty$  = surrounding fluid temperature

**Example 1.** A 0.0394 in. diameter copper sphere is to be used as a sensing element for a thermostat. It is initially at a uniform temperature of 69.8°F. It is then exposed to the surrounding air at 68°F. The combined heat transfer coefficient is  $h = 10.63$  Btu/h·ft<sup>2</sup>·°F. Determine the time taken for the temperature of the sphere to reach 69.6°F. The properties of copper are

$$\rho = 557.7 \text{ lb}_m/\text{ft}^3 \quad c_p = 0.0920 \text{ Btu/lb}_m \cdot \text{°F} \quad k = 232 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F}$$

$Bi = hR/k = 10.63(0.0394/12/2)/232 = 7 \times 10^{-5}$ , which is much less than 1. Therefore, lumped analysis is valid.

$$M = \rho(4\pi R^3/3) = 10.31 \times 10^{-6} \text{ lb}_m$$

Using Equation (8),  $\tau = 2.778 \times 10^{-4} \text{ h} = 1 \text{ s}$ .

**Nonlumped Analysis.** In cases where the Biot number is greater than 0.1, the variation of temperature with location within the mass must be accounted for. This requires solving multidimensional partial differential equations. Many common cases have been solved and presented in graphical forms (Jakob 1957; Myers 1971; Schneider 1964). In other cases, it is simpler to use numerical methods (Croft and Lilley 1977; Patankar 1980). When convective boundary conditions are required in the solution, values of  $h$  based on steady-

**Table 2 Conduction Shape Factors**

Configuration	Shape Factor $S$ , ft	Restriction	
Edge of two adjoining walls	0.54W	$W > L/5$	
Corner of three adjoining walls (inner surface at $T_1$ and outer surface at $T_2$ )	0.15L	$L \ll$ length and width of wall	
Isothermal rectangular block embedded in semi-infinite body with one face of block parallel to surface of body	$\frac{2.756L}{\left[\ln\left(1 + \frac{d}{W}\right)\right]^{0.59}} \left(\frac{H}{d}\right)^{0.078}$	$L > W$ $L \gg d, W, H$	
Thin isothermal rectangular plate buried in semi-infinite medium	$\frac{\pi W}{\ln(4W/L)}$ $\frac{2\pi W}{\ln(4W/L)}$ $\frac{2\pi W}{\ln(2\pi d/L)}$	$d = 0, W > L$ $d \gg W$ $W > L$ $d > 2W$ $W \gg L$	
Cylinder centered inside square of length $L$	$\frac{2\pi L}{\ln(0.54W/R)}$	$L \gg W$ $W > 2R$	
Isothermal cylinder buried in semi-infinite medium	$\frac{2\pi L}{\cosh^{-1}(d/R)}$ $\frac{2\pi L}{\ln(2d/R)}$ $\frac{2\pi L}{\ln\left[\frac{L}{R}\left(1 - \frac{\ln(L/2d)}{\ln(L/R)}\right)\right]}$	$L \gg R$ $L \gg R$ $d > 3R$ $d \gg R$ $L \gg d$	
Horizontal cylinder of length $L$ midway between two infinite, parallel, isothermal surfaces	$\frac{2\pi L}{\ln\left(\frac{4d}{R}\right)}$	$L \gg d$	
Isothermal sphere in semi-infinite medium	$\frac{4\pi R}{1 - (R/2d)}$		
Isothermal sphere in infinite medium	4πR		

state correlations are often used. However, this approach may not be valid when rapid transients are involved.

*Estimating Cooling Times for One-Dimensional Geometries.* Cooling times for materials can be estimated (McAdams 1954) by Gurnie-Lurie charts (Figures 2, 3, and 4), which are graphical solutions for heating or cooling of constant cross-sectional area slabs with heat transfer from only two parallel surfaces, solid cylinders with heat transfer from only the cylindrical surface, and solid spheres. These charts assume an initial uniform temperature distribution and no change of phase. They apply to a body exposed to a constant-temperature fluid with a constant surface heat transfer coefficient of  $h$ .

Using Figures 2, 3, and 4 for constant cross-sectional area solids, long solid cylinders, and spheres, it is possible to estimate both the temperature at any point and the average temperature in a homogeneous mass of material as a function of time in a cooling process. The charts can also be used for fluids, which, absent motion, behave as solids.

Figures 2, 3, and 4 represent four dimensionless quantities: temperature  $Y$ , distance  $n$ , inverse of the Biot number  $m$ , time, and  $Fo$ , as follows:

$$Y = \frac{t_c - t_2}{t_c - t_1} \quad n = \frac{r}{r_m} \quad m = \frac{k}{hr_m} \quad Fo = \frac{\alpha\tau}{r_m^2}$$

where

- $t_c$  = temperature of fluid medium surrounding solid
- $t_1$  = initial uniform temperature of solid
- $t_2$  = temperature at a specified location  $n$ , and dimensionless time  $Fo$
- $r$  = distance from midplane (constant cross-sectional area slab), or radius (solid cylinder and sphere)
- $r_m$  = half-thickness (constant cross-sectional area slab), or outer radius (solid cylinder and sphere)
- $k$  = thermal conductivity of solid
- $\alpha$  = thermal diffusivity of solid =  $k/\rho c_p$
- $\rho$  = density of solid
- $c_p$  = constant pressure specific heat of solid
- $\tau$  = time
- $Fo$  = Fourier number

*Approximate Solution for  $Fo > 0.2$ .* For  $Fo > 0.2$ , the transient temperature is very well approximated by the first term of the series solution. The single-term approximations for the three cases are:

**Constant cross-sectional area solid (thickness  $2L$ ):**

$$Bi = \frac{hL}{k} \quad Fo = \frac{\alpha\tau}{L^2}$$

$$Y = c_1 \exp(-\mu_1^2 Fo) \cos(\mu_1 n) \quad c_1 = \frac{2 \sin(\mu_1)}{\mu_1 + \sin(\mu_1) \cos(\mu_1)} \quad (9)$$

**Solid cylinder (radius  $R$ ):**

$$Bi = \frac{hR}{k} \quad Fo = \frac{\alpha\tau}{R^2}$$

$$Y = c_1 \exp(-\mu_1^2 Fo) J_0(\mu_1 n) \quad c_1 = \frac{2 Bi \sin(\mu_1)}{(\mu_1^2 + Bi^2) J_0(\mu_1)} \quad (10)$$

where  $J_0$  is the Bessel function of the first kind, order zero.

**Solid sphere (radius  $R$ ):**

$$Bi = \frac{hR}{k} \quad Fo = \frac{\alpha\tau}{R^2}$$

$$Y = \frac{c_1 \exp(-\mu_1^2 Fo) \sin(\mu_1 n)}{\mu_1 n} \quad c_1 = \frac{2 Bi \sin(\mu_1)}{\mu_1 - \sin(\mu_1) \cos(\mu_1)} \quad (11)$$

where  $c_1$  and  $\mu_1$  are coefficients in the first term of the series solutions.

Values of  $c_1$  and  $\mu_1$  are given in Table 3 for a few values of  $Bi$ .

From a regression analysis (Couvillion 2004), the values of  $\mu_1$  in Table 3 can be expressed as

$$\frac{1}{\mu_1} = a_0 + \frac{a_1}{Bi} + a_2 \exp\left(-\frac{a_3}{Bi}\right) \quad (12)$$

where the values of curve fit constants  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$  are given in Table 4.

Finding  $\mu_1$  allows calculation of the constant  $c_1$  from Equations (9), (10), or (11). Note that  $\mu_1$  is in radians. The Bessel functions in Equation (10) are available in many software applications.

**Example 2.** Apples, approximated as 2.36 in. diameter solid spheres and initially at 86°F, are loaded into a chamber maintained at 32°F. If the surface heat transfer coefficient is 2.47 Btu/h·ft<sup>2</sup>·°F, estimate the time required for the center temperature to reach 33.8°F.

Properties of apples are

$$\rho = 51.8 \text{ lb}_m/\text{ft}^3 \quad k = 0.243 \text{ Btu/h}\cdot\text{ft}\cdot\text{°F}$$

$$c_p = 0.860 \text{ Btu/lb}_m\cdot\text{°F} \quad d = 2.36 \text{ in.} = 0.1967 \text{ ft}$$

$$h = 2.47 \text{ Btu/h}\cdot\text{ft}^2\cdot\text{°F}$$

Assuming that it will take a long time for the center temperature to reach 33.8°F, use the one-term approximation Equation (11). From the values given,

$$Y = \frac{t_c - t_2}{t_c - t_1} = \frac{32 - 33.8}{32 - 86} = 0.0333 \quad n = \frac{r}{r_m} = \frac{0}{0.1967} = 0$$

$$Bi = \frac{hr_m}{k} = \frac{2.47 \times (0.1967/2)}{0.243} = 1$$

$$\alpha = \frac{k}{\rho c_p} = \frac{0.243}{51.8 \times 0.860} = 0.00545 \text{ ft}^2/\text{h}$$

From Equation (11) with  $\lim(\sin 0/0) = 1$ ,  $Y = c_1 \exp(-\mu_1^2 Fo)$ . For  $Bi = 1$ , from Table 3,  $c_1 = 1.2732$  and  $\mu_1 = 1.5708$ . Thus,

$$Fo = -\frac{1}{\mu_1^2} \ln \frac{Y}{c_1} = -\frac{1}{1.5708^2} \ln 0.0333 = 1.476 = \frac{\alpha\tau}{r_m^2} = \frac{0.00545\tau}{(0.1967/2)^2}$$

$$\tau = 2.62 \text{ h}$$

Note that using Equation (12) gives a value of 1.57 for  $\mu_1$ , which is only 0.05% less than the tabulated value of 1.5708. Also note that  $Fo = 0.2$  corresponds to an actual time of 0.36 h.

Although the one-term approximation for temperature distribution is satisfactory for  $Fo > 0.2$ , the total heat transferred from  $Fo = 0$  to  $Fo = 0.2$  as a fraction of the heat transferred from  $Fo = 0$  to  $Fo = \infty$  becomes significant as  $Bi$  increases. In those cases, calculating additional terms in the series solution may be needed for satisfactory results.

*Multidimensional Temperature Distribution.* One-dimensional transient temperature distribution charts can be used to find the temperatures with two- and three-dimensional temperatures of solids. For example, consider a solid cylinder of length  $2L$  and radius  $r_m$  exposed to a fluid at  $t_c$  on all sides with constant surface heat transfer coefficients  $h_1$  on the end surfaces and  $h_2$  on the cylindrical surface, as shown in Figure 5.

The two-dimensional, dimensionless temperature  $Y(x_1, r_1, \tau)$  can be expressed as the product of two one-dimensional temperatures  $Y_1(x_1, \tau) \times Y_2(r_1, \tau)$ , where

- $Y_1$  = dimensionless temperature of constant cross-sectional area slab at  $(x_1, \tau)$ , with surface heat transfer coefficient  $h_1$  associated with two parallel surfaces
- $Y_2$  = dimensionless temperature of solid cylinder at  $(r_1, \tau)$  with surface heat transfer coefficient  $h_2$  associated with cylindrical surface

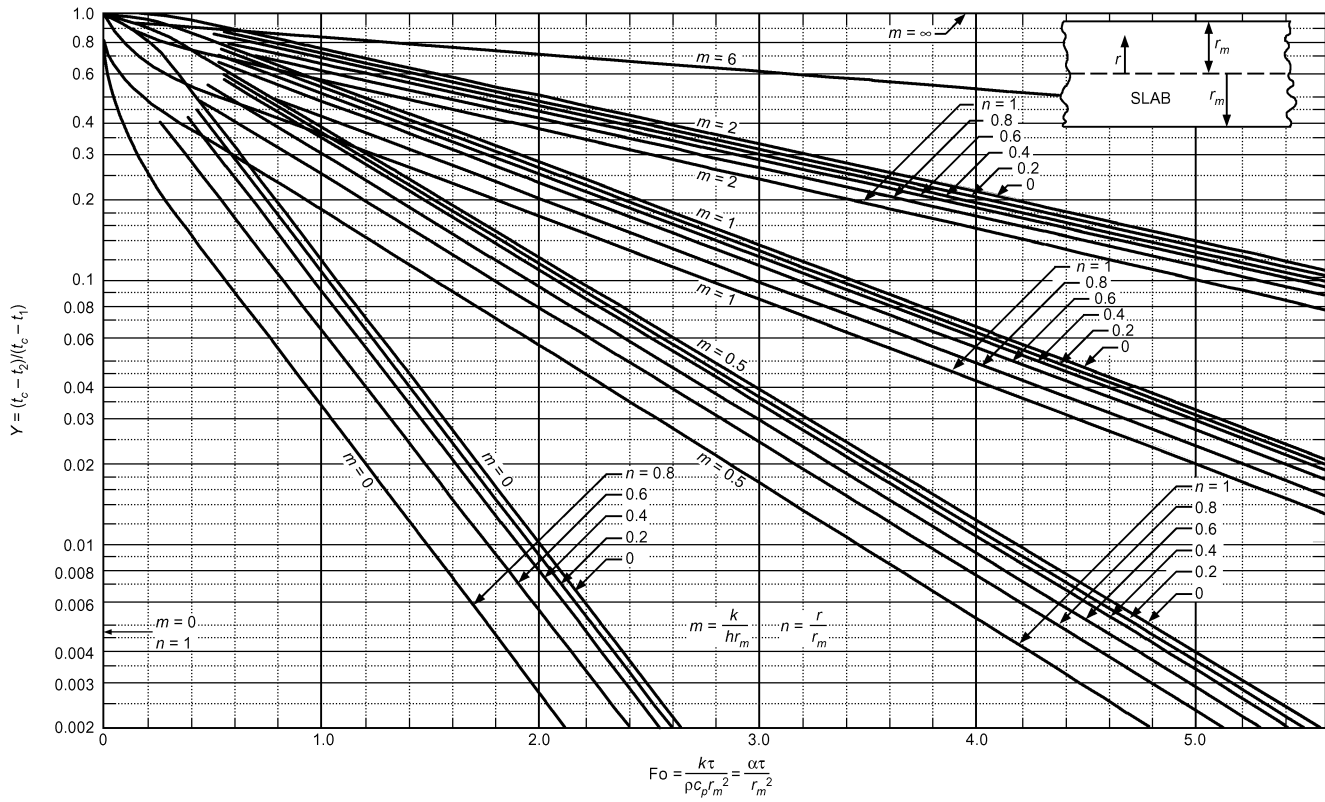


Fig. 2 Transient Temperatures for Infinite Slab

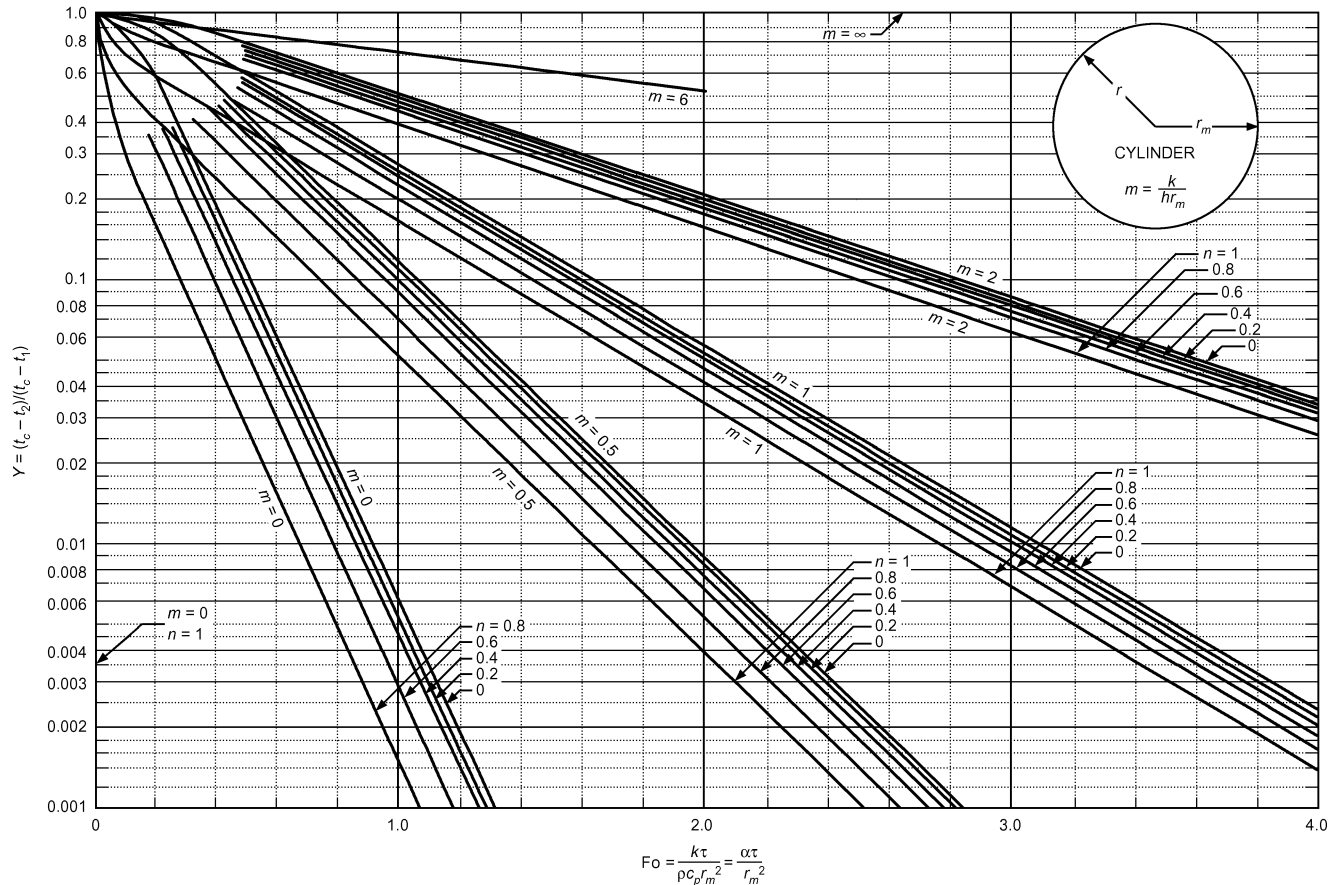


Fig. 3 Transient Temperatures for Infinite Cylinder